Basic Quadrilateral Proofs

For each of the following, draw a diagram with labels, create the givens and proof statement to go with your diagram, then write a two-column proof. Make sure your work is neat and organized.

Quadrilateral Proof:

1. Prove that the sum of the interior angles of a quadrilateral is 360^o.

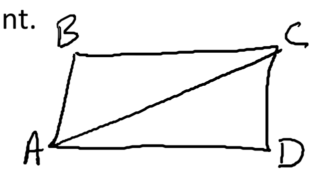
Given: Quadrilateral $ABCD$

Prove: $m\angle A + m\angle B + m\angle C + m\angle D = 360^o$

Statement	Reason
1. Quadrilateral $ABCD$	1. Given
2. $m\angle BAC + m\angle B + m\angle BCA = 180^o$ $m\angle CAD + m\angle D + m\angle ACD = 180^o$	2. The sum of the interior angles of a triangles add up to 180^o
3. $m\angle BAC + m\angle B + m\angle BCA + m\angle CAD + m\angle D + m\angle ACD = 360^o$	3. Addition and Substitution postulates
4. $m\angle A = m\angle BAC + m\angle CAD$ $m\angle C = m\angle BCA + m\angle ACD$	4. Angle addition postulate
5. $m\angle A + m\angle B + m\angle C + m\angle D = 360^o$	5. Substitution postulate (steps 5,6)

Parallelogram Proofs:

2. Prove the opposite sides of a parallelogram are congruent.

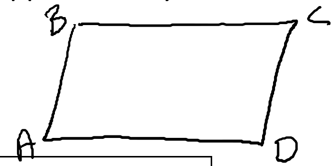


Given: Parallelogram $ABCD$

Prove: $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$

Statement	Reason
1. Parallelogram $ABCD$	1. Given
2. $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$	2. Definition of parallelogram
3. $\angle BAC \cong \angle DCA$ and $\angle DAC \cong \angle BCA$	3. Alternate Interior angles are $\cong$
4. $\overline{AC} \cong \overline{AC}$	4. Reflexive property of $\cong$
5. $\Delta ABC \cong \Delta CDA$	5. ASA $\cong$
6. $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$	6. CPCTC

3. Prove that any pair of consecutive angles of a parallelogram are supplementary.



Given: Parallelogram $ABCD$

Prove: $\angle A$ and $\angle B$ are supplementary

Statement	Reason
1. Parallelogram $ABCD$	1. Given
2. $\overline{BC} \parallel \overline{AD}$	2. Definition of parallelogram
3. $\angle A$ and $\angle B$ are supplementary	3. Same side interior angles are supplementary.

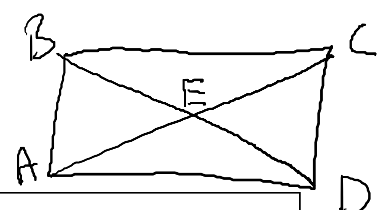
4. Prove that opposite angles of a parallelogram are congruent.

Given: Parallelogram $ABCD$

Prove: $\angle A \cong \angle C$ and $\angle B \cong \angle D$

Statement	Reason
1. Parallelogram $ABCD$	1. Given
2. $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$	2. Definition of parallelogram
3. $\angle BAC \cong \angle DCA$ and $\angle DAC \cong \angle BCA$	3. Alternate Interior angles are $\cong$
4. $\overline{AC} \cong \overline{AC}$	4. Reflexive property of $\cong$
5. $\Delta ABC \cong \Delta CDA$	5. ASA $\cong$
6. $\angle B \cong \angle D$	6. CPCTC
7. $\angle A$ and $\angle B$ are supplementary $\angle C$ and $\angle D$ are supplementary	7. Property of parallelogram proved in problem #3
8. $\angle B \cong \angle D$	8. Supplements of $\cong$ angles are $\cong$ (steps 6,7)

5. Prove that the diagonals of a parallelogram bisect each other.



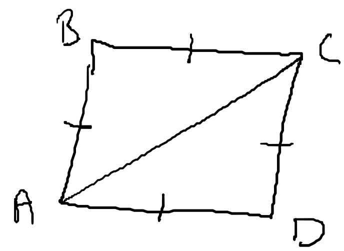
Given: Parallelogram $ABCD$

Prove: $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$

Statement	Reason
1. Parallelogram $ABCD$	1. Given
2. $\overline{AB} \parallel \overline{CD}$	2. Definition of parallelogram
3. $\angle ABE \cong \angle CDE$ $\angle BAE \cong \angle DCE$	3. Alternate Interior Angles are $\cong$ (step 2)
4. $\overline{AB} \cong \overline{CD}$	4. Property of parallelogram proved in #2
5. $\Delta AEB \cong \Delta CED$	5. ASA $\cong$
6. $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$	6. CPCTC

Rhombus Proofs:

6. Prove that a rhombus is a parallelogram.

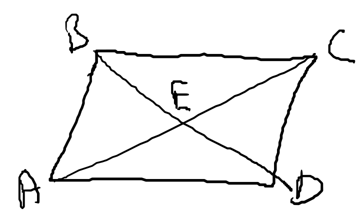


Given: Rhombus $ABCD$

Prove: $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$

Statement	Reason
1. Rhombus $ABCD$	1. Given
2. $\overline{AB} \cong \overline{CD} \cong \overline{BC} \cong \overline{AD}$	2. Definition of rhombus
3. $\overline{AC} \cong \overline{AC}$	3. Reflexive property of $\cong$
4. $\Delta ABC \cong \Delta CDA$	4. SSS $\cong$
5. $\angle BAC \cong \angle DCA$	5. CPCTC
6. $\overline{AB} \parallel \overline{CD}$	6. Converse of alternate-interior angles
7. $\angle CAD \cong \angle ACB$	7. CPCTC
8. $\overline{BC} \parallel \overline{AD}$	8. Converse of alternate-interior angles

7. Prove that the diagonals of a rhombus are perpendicular.

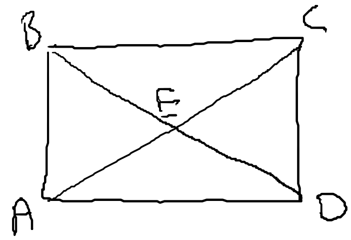


Given: Rhombus $ABCD$

Prove: $\overline{AC} \perp \overline{BD}$

Statement	Reason
1. Rhombus $ABCD$	1. Given
2. $\overline{AB} \cong \overline{BC}$	2. Definition of rhombus
3. $ABCD$ is a parallelogram	3. Property of a rhombus proved in #6
4. $\overline{AE} \cong \overline{CE}$	4. Property of a parallelogram proved in #5
5. $\Delta ABE \cong \Delta CBE$	5. SSS $\cong$
6. $\angle AEB \cong \angle CEB$	6. CPCTC
7. $m\angle AEB = m\angle CEB$	7. Definition of $\cong$ segment
8. $\angle AEB$ and $\angle CEB$ are supplementary	8. Two angles that form a linear pair are supplementary
9. $m\angle AEB + m\angle CEB = 180^o$	9. Definition of supplementary angles
10. $m\angle AEB + m\angle AEB = 180^o$	10. Substitution postulate (steps 7,9)
11. $m\angle AEB = 90^o$	11. Division postulate
12. $\angle AEB$ is a right angle	12. Definition of right angle
13. $\overline{AC} \perp \overline{BD}$	13. Definition of perpendicular segments

8. Prove that each diagonal of a rhombus bisects a pair of opposite angles.



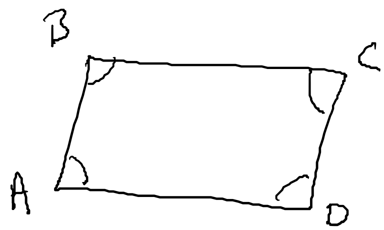
Given: Rhombus $ABCD$

Prove: $\angle BAE \cong \angle DAE, \angle BCE \cong \angle DCE, \angle ABE \cong \angle CBE, \angle ADE \cong \angle CDE$

Statement	Reason
1. Rhombus $ABCD$	1. Given
2. $\overline{AB} \cong \overline{CD} \cong \overline{BC} \cong \overline{AD}$	2. Definition of rhombus
3. $ABCD$ is a parallelogram	3. Property of rhombus proved in #6
4. $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$	4. Property of parallelogram proved in #5
5. $\Delta AEB \cong \Delta CEB \cong \Delta CED \cong \Delta AED$	5. SSS $\cong$
6. $\angle BAE \cong \angle DAE, \angle BCE \cong \angle DCE,$ $\angle ABE \cong \angle CBE, \angle ADE \cong \angle CDE$	6. CPCTC

Rectangles Proofs:

9. Prove that a rectangle is a parallelogram.

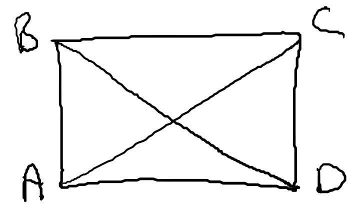


Given: Rectangle $ABCD$

Prove: $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$

Statement	Reason
1. Rectangle $ABCD$	1. Given
2. $\angle A \cong \angle B \cong \angle C \cong \angle D$	2. Definition of rectangle
3. $m\angle A = m\angle B = m\angle C = m\angle D$	3. Definition of $\cong$ angles
4. $m\angle A + m\angle B + m\angle C + m\angle D = 360^o$	4. Property of Quadrilateral proved in #1
5. $4 \cdot m\angle A = 360^o$	5. Substitution postulate (steps 3,7)
6. $m\angle A = 90^o$	6. Division postulate
7. $m\angle B = m\angle C = m\angle D = 90^o$	7. Substitution postulate (steps 3,9)
8. $m\angle A + m\angle B = 180^o$ $m\angle A + m\angle D = 180^o$	8. Substitution postulate
9. $\angle A$ and $\angle B$ are supplementary $\angle A$ and $\angle D$ are supplementary	9. Definition of supplementary angles
10. $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$	10. Converse of same-side interior

10. Prove that the diagonals of a rectangle are congruent.



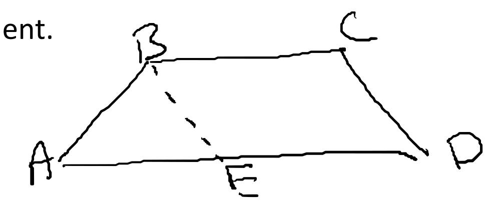
Given: Rectangle $ABCD$

Prove: $\overline{AC} \cong \overline{BD}$

Statement	Reason
1. Rectangle $ABCD$	1. Given
2. $\angle A \cong \angle D$	2. Definition of rectangle
3. $\overline{AD} \cong \overline{AD}$	3. Reflexive Property
4. $ABCD$ is parallelogram	4. Property of rectangle proved in #9
5. $\overline{AB} \cong \overline{CD}$	5. Property of parallelogram proved in #2
6. $\Delta BAD \cong \Delta CDA$	6. SAS $\cong$
7. $\overline{AC} \cong \overline{BD}$	7. CPCTC

Trapezoid Proofs:

11. Prove each pair of base angles of an isosceles trapezoid is congruent.

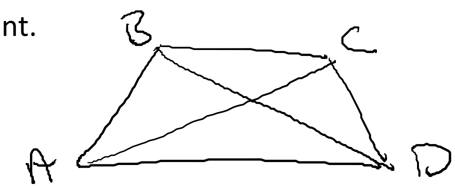


Given: Isosceles Trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$

Prove: $\angle A \cong \angle D$ and $\angle ABC \cong \angle C$

Statement	Reason
1. Isosceles Trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$	1. Given
2. $\overline{AB} \cong \overline{CD}$	2. Definition of Isosceles Trapezoid
3. Construct $\overline{BE} \parallel \overline{CD}$	3. Construction
4. Parallelogram $EBCD$	4. Definition of parallelogram (steps 1,3)
5. $\overline{BE} \cong \overline{CD}$	5. Property of parallelogram proved in #2
6. $\overline{AB} \cong \overline{BE}$	6. Transitive property of $\cong$ (steps 2,5)
7. $\angle A \cong \angle AEB$	7. Isosceles triangle theorem
8. $\angle AEB \cong \angle D$	8. Corresponding angles are $\cong$
9. $\angle A \cong \angle D$	9. Transitive property
10. $\angle A$ and $\angle ABC$ are supplementary $\angle C$ and $\angle D$ are supplementary	10. Same side interior angles are supplementary
11. $\angle ABC \cong \angle C$	11. Supplements of $\cong$ angles are $\cong$ (steps 9,10)

11. Prove that the diagonals of an isosceles trapezoid are congruent.



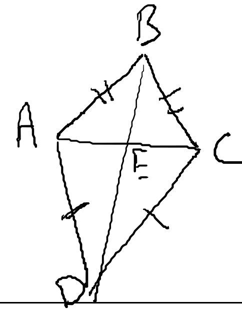
Given: Isosceles Trapezoid $ABCD$

Prove: $\overline{AC} \cong \overline{BD}$

Statement	Reason
1. Isosceles Trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$	1. Given
2. $\overline{AB} \cong \overline{CD}$	2. Definition of Isosceles Trapezoid
3. $\angle BAD \cong \angle CDA$	3. Property of Isosceles Trapezoid proved in #11
4. $\overline{AD} \cong \overline{AD}$	4. Reflexive Property
5. $\Delta BAD \cong \Delta CDA$	5. SAS $\cong$
6. $\overline{AC} \cong \overline{BD}$	6. CPCTC

Kite Proofs:

12. Prove that the diagonals of a kite are perpendicular.



Given: Kite $ABCD$

Prove: $\overline{AC} \perp \overline{BD}$

Statement	Reason
1. Kite $ABCD$	1. Given
2. $\overline{AB} \cong \overline{BC}, \overline{AD} \cong \overline{CD}$	2. Definition of Kite
3. $\angle BAC \cong \angle BCA$	3. Isosceles Triangle theorem
4. $\overline{BD} \cong \overline{BD}$	4. Reflexive property of $\cong$
5. $\Delta DAB \cong \Delta DCB$	5. SSS $\cong$ (steps 2,4)
6. $\angle ABD \cong \angle CBD$	6. CPCTC
7. $\Delta ABE \cong \Delta CBE$	7. ASA $\cong$ (steps 2,3,6)
8. $\angle AEB \cong \angle CEB$	8. CPCTC
9. $\angle AEB$ and $\angle CEB$ are supplementary	9. Two angles that form a linear pair are supplementary
10. $m\angle AEB + m\angle CEB = 180^o$	10. Definition of supplementary
11. $m\angle AEB = m\angle CEB$	11. Definition of $\cong$ angles (step 8)
12. $m\angle AEB + m\angle AEB = 180^o$	12. Substitution property (steps 10,11)
13. $m\angle AEB = 90^o$	13. Division property
14. $\overline{AC} \perp \overline{BD}$	14. Definition of perpendicular segments